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**B.M.S COLLEGE FOR WOMEN AUTONOMOUS**
**BENGALURU-560004**
**END SEMESTER EXAMINATION – APRIL /MAY 2023**

## M.Sc. Mathematics – I Semester

# ORDINARY DIFFERENTIAL EQUATIONS

**Course Code MM104T** **QP Code: 11004**
**Duration: 3 Hours** **Maximum Marks: 70**

Instructions: 1) **All** questions carry **equal** marks.
2) Answer **any five** full questions.

1. a) Find the Wronskian of $y^{(5)} - y^{(4)} - y' + y = 0$ on $I = [0,1]$.
   b) Define a fundamental set. Show that $\{\Psi_j(x): j = 1 \dots . n\}$ forms a fundamental set for $L_n y = 0$ on $I$ if and only if $W\{\Psi_j(x): j = 1 \dots . n\} \neq 0, \forall\, x \in I$.
   c) If $\phi_1(x)$ is a solution of $y'' + a_1(x)y' + a_2(x)y = 0$. Then show that $\phi_2(x) = \phi_1(x)f(x)$ is also a solution of the same differential equation provided $f'(x)$ satisfies the equation $(\phi_1^2 y)' + a_1(\phi_1^2 y) = 0$. Also show that $\phi_1(x)$ and $\phi_2(x)$ are linearly independent. **(3+7+4)**

2. a) State and prove Lagrange's identity.
   b) Solve : $y'' + y = cosec\, x,\ 0 < x < \frac{\pi}{2}\ ,\ y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$ by the method of variation of parameters.
   c) Define adjoint differential equation. Find the solution of $x^2y'' + 9xy' + 12y = 0$ by finding the solution of adjoint equation. **(6+4+4)**

3. a) State and prove Sturm's comparison theorem on the zeros of the solution of a self-adjoint differential equations.
   b) If $\{\phi_j(x): j = 1 \dots . n\}$ be solutions of $L_n y = 0$ on some interval $I$ , then prove that $W\{\phi_j(x): j = 1 \dots . n\} = W\{\phi_j(x_0): j = 1 \dots . n\} * exp\left\{- \int_{x_0}^{x} \frac{a_1(t)}{a_0(t)} dt\right\}, x, x_0 \in I.$
   **(7+7)**

4. a) Find the Eigen value and Eigen functions of the differential equation
   $y'' + \lambda y = 0;\ y(0) = 0 = y(\pi).$
   b) Define a self-adjoint eigen value problem. For such a problem, prove that
   (i) The eigenvalues are real.

**(ii)** The eigen functions corresponding to distinct eigen values are orthogonal over given interval. **(7+7)**

5. a) Find the ordinary points, singular points, regular-singular points, irregular singular points and points at infinity of the Chebyshev differential equation.

   b) Using Frobenius method obtain the solution of the differential equation $2x^2y'' + xy' + (x^2 - 3)y = 0$ about its regular singular point. **(5+9)**

6. a) Prove that: $\frac{1}{1-t}\, exp\left(\frac{-xt}{1-t}\right) = \sum_{n=0}^{\infty} L_n(x)t^n$.

   b) Prove the orthogonal property for Chebyshev polynomials.

   c) Prove the following relations:

   (i) $H_n'(x) = 2nH_{n-1}(x)$ (ii) $H_n''(x) - 2xH_n'(x) + 2nH_n(x) = 0$. **(5+5+4)**

7. a) Find the fundamental matrix solution of the following system of equations:

$$\frac{dX}{dt} = A(t)X(t),\ \ A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -4 \end{bmatrix},\ X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

   b) Explain various types of critical points of the linear system:

$$\frac{dx}{dt} = ax + by\ ;\ \frac{dy}{dt} = cx + d,\quad ad - bc \neq 0.$$ **(7+7)**

8. a) Find all the critical points, their nature and stability of each of the critical point of the given system: $\frac{dx}{dt} = 1 - y;\ \frac{dy}{dt} = x^2 - y^2$

   b) State Liapunov theorem. Construct Liapunov function and hence determine the stability of the critical point (0,0) of the following nonlinear systems:

   (i) $\frac{dx}{dt} = -x + 2x^2 + y^2\ ;\ \frac{dy}{dt} = -y + xy$

   (ii) $\frac{dx}{dt} = x^3 - 3xy^4\ ;\ \frac{dy}{dt} = x^2y - 2y^3 - y^5$ **(7+7)**

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